ICM University of Warsaw

Seminar on

Nonlinear Mathematical Analysis with Applications

ICM, University of Warsaw, Pawinskiego 5a, 02-106 Warsaw

Maria Gokeili, Nobuyuki Kenmochi, Marek Niezgodka

Information on Lectures

(ICM,UW, 5 floor, Pawinskiego 5a, Warsaw)

June 5, 2017: 15:00 - 15:30

Prof. Ken Shirakawa (Chiba Univ., Japan)

Title: Phase field systems of grain boundary motions with time-dependent Dirichlet boundary data

Abstract This study is based on recent jointworks with Dr. J. Salvador Moll (Univ. Valencia, Spain) and Prof. Hiroshi Watanabe (Oita Univ., Japan).

Let $\Omega \subset \mathbb{R}^N$, with $N \in \mathbb{N}$, be a bounded domain, having sufficiently smooth boundary Γ and the unit outer normal n_Γ. On this basis, we consider the following system of PDEs:

$$\begin{cases} \partial_t \eta - \varDelta \eta + g'(\eta) + \alpha'(\eta)|D\theta| = 0, \text{ in } Q := (0,\infty) \times \Omega, \\ \nabla \eta \cdot n_\Gamma + \alpha'(\eta)|\theta - f(t,x)| = 0, \ (t,x) \in \Sigma := (0,\infty) \times \Gamma, \end{cases} \tag{1}$$

$$\begin{cases} \alpha_0(\eta)\partial_t \theta - \mathrm{div}\left(\alpha(\eta)\dfrac{D\theta}{|D\theta|}\right) \ni 0, \text{ in } Q := (0,\infty) \times \Omega, \\ \partial I_{[-1,1]}\left(-\frac{D\theta}{|D\theta|} \cdot n_\Gamma\right) \ni \theta - f(t,x), \ (t,x) \in \Sigma, \end{cases} \tag{2}$$

subject to suitable initial conditions. This system is based on the phase-field system of grain boundary motion, proposed in [Kobayashi et al.,Phys. D, 140, 141–150 (2000)], and is derived as a gradient system of the following *free-energy:*

$$[\eta,\theta] \in H^1(\Omega) \times BV(\Omega) \mapsto \mathcal{F}(\eta,\theta) := \frac{1}{2}\int_\Omega |\nabla \eta|^2 \, dx + \int_\Omega g(\eta)\, dx + \int_\Omega \alpha(\eta)|D\theta| + \int_\Gamma \alpha(\eta)|\theta - f(t,x)|\, d\Gamma. \tag{3}$$

In the context, the unknowns η and θ are order parameters to indicate, respectively, the orientation order and orientation angle, in a polycrystal. $f = f(t,x)$ is a time-dependent and inhomogeneous boundary data, and α_0, α and g are given functions. "$'$" denotes the differential of function, and $\partial I_{[-1,1]}$ denotes the subdifferential of the indicator function $I_{[-1,1]}$ on the closed interval $[-1,1]$.

One of the characteristics of our system is in: the non-standard Neumann-type boundary condition in (1); and the non-standard Dirichlet-type boundary condition in (2); which are brought by the interacting integral $\int_\Gamma \alpha(\eta)|\theta - f(t,x)|\, d\Gamma$ as in (3). Referring to some previous works, e.g. [Andreu et al., J. Funct. Anal., 180 (2001), no. 2, 347–403.] and [Moll et. al., Nonlinearity, 30 (2017), 2752–2789], we establish the mathematical method to deal with the non-standards, and set the goal to demonstrate the mathematical results concerned with the following issues.

(A) Large-time behavior to show the association between the ω-limit points of orbits as $t \to \infty$, and the solutions to the steady-state system for $\{(1)–(2)\}$.

(B) Structure of steady-state solutions to observe the profiles of steady-state solutions in some specific cases, e.g. one-dimensional case.